2014 released AP Free Response Solutions

The questions are available at:

AB: http://media.collegeboard.com/digitalServices/pdf/ap/ap14_frq_calculus_ab.pdf

BC: http://media.collegeboard.com/digitalServices/pdf/ap/ap14_frq_calculus_bc.pdf

There are several different forms used around the country. These are the only questions that can be discussed from 2014.

AB1+BC1) $A(t) = 6.687(0.931)^t$ where $A(t)$ is in pounds; t is in days; domain is [0, 30].

1a-AB-BC) Recall – average rate of change is the *slope* of the *secant line*. So

avg r.o.c. = $\dfrac{A(30)-A(0)}{30-0} = \dfrac{0.7829278694-6.687}{30} = -0.1968024044$ pounds per day

1b-AB-BC) $A'(t) = 6.687(0.931)^t * \ln(0.931)$. So

$A'(15) = 6.687(0.931)^{15} * \ln(0.931) = -0.1635905804$. What this means is that, at the beginning of the 15th day, the amount of grass clippings remaining in the bin is decreasing at a rate of 0.1636 pounds per day.

1c-AB-BC) Average grass clippings in the bin = average value of $A(t)$ over the interval [0, 30] =

$\dfrac{1}{30-0}\int_0^{30} A(t)dt \approx \dfrac{1}{30}\left(82.57905323\right) = 2.752635108$.

To find the time t where $A(t)$ = 2.752635108, solve and get $t = 12.414774$

1d-AB-BC) $A'(t) = 6.687 * \ln(0.931) * (0.931)^t$.

$A'(30) = 6.687 * \ln(0.931) * (0.931)^{30} \approx -0.0559762123$

$L(t) = A(30) + (-0.0559762123)(t-30)$

$= 0.7829278694 - 0.0559762123(t-30)$

Solve $L(t) = 0.5$ and so $t = 35.0544304$ days.

One possible scoring: 1a) +1 for answer; +1 units in 1a and 1b

1b) +1 derivative; +1 explanation

1c) +1 average value; +1 A(t)=average value; +1 t=

1d) +1 L(t); +1 t=

AB2) R is enclosed by $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$. The points of intersection are: $(x, y) = (0, 4)$ and $(x, y) = (2.3, 4)$.

2a-AB) $Volume = \pi\int_0^{2.3} (4-(-2))^2 - \left(x^4 - 2.3x^3 + 4 - (-2)\right)^2 dx = 98.86788997$

2b-AB) $Volume = \int_0^{2.3} \left[\frac{1}{2}\left(4-\left(x^4 - 2.3x^3 + 4\right)\right)^2\right] dx = 3.573715598$

2c-AB) $\int_0^k 4-\left(x^4 - 2.3x^3 + 4\right)dx = \int_k^{2.3} 4-\left(x^4 - 2.3x^3 + 4\right)dx$

One possible scoring: 2a) +1 limits; +1 integrand; +1 answer
2b) +1 limits; +1 integrand; +1 answer
2c) +2 limits; +1 integrands

BC2) Polar curves: $r = 3$ and $r = 3 - 2\sin(2\theta)$. Domain = $[0, 2\pi]$

Points of intersection: $(r,\theta) = (3,0), \left(3, \frac{\pi}{2}\right), (3,\pi)$

2a-BC) $Area = \int_0^{\pi/2} \frac{(3-2\sin(2\theta))^2}{2} d\theta + \int_{\pi/2}^{\pi} \frac{3^2}{2} d\theta = 9.707963268$

2b-BC) $\begin{aligned} x(\theta) &= r\cos\theta = \left(3 - 2\sin(2\theta)\right)\cos(\theta) = 3\cos(\theta) - 2\sin(2\theta)\cos(\theta) \\ x'(\theta) &= -3\sin(\theta) - 2\left[2\cos(2\theta)\cos(\theta) + \sin(2\theta)\left(-\sin(\theta)\right)\right] \end{aligned}$. So

$\frac{dx}{d\theta}\Big|_{\theta=\pi/6} = -3\sin(\pi/6) - 2\left[2\cos\left(2(\pi/6)\right)\cos(\pi/6) + \sin\left(2(\pi/6)\right)\left(-\sin(\pi/6)\right)\right] =$

$-\frac{3}{2} - \sqrt{3} + \frac{\sqrt{3}}{2} = -\frac{3}{2} - \frac{\sqrt{3}}{2} \approx -2.3660254038$

2c-BC) $\text{distance} = 3 - \left(3 - 2\sin(2\theta)\right) = 2\sin(2\theta)$

$\frac{d}{d\theta}\left(2\sin(2\theta)\right) = 4\cos(2\theta)$. So $4\cos(2\theta)\big|_{\theta=\pi/3} = 4\cos\left(\frac{2\pi}{3}\right) = -2$

2d-BC) $\frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = \left(-4\cos(2\theta)\right)\cdot(3) = -12\cos(2\theta)$. So, at $\theta = \frac{\pi}{6}$, $\frac{dr}{dt} = -12\cos\left(\frac{\pi}{3}\right) = -6$

One possible scoring: 2a) +1 limits, +1 integrand, +1 answer
2b) +1 x'; +1 answer
2c) +1 distance; +1 derivative; +1 answer
2d) +1 answer

AB3+BC3) $g(x)=\int_{-3}^{x} f(t)dt$. $f(t)$ given by a graph.

3a-AB-BC) $g(3)=\int_{-3}^{3} f(t)dt = \text{net area between } f(t) \text{ and x-axis}$. The net area is + the area of the triangle from t=-3 to t=2 and then MINUS the area of the triangle from 2 to 3.

So $g(3)=\frac{1}{2}(5)(4)-\frac{1}{2}(1)(2)=10-1=9$

3b-AB-BC) The first derivative of *g* must be positive on an interval for the graph of *g* to be increasing. The second derivative of *g* must be negative on an interval for the graph of *g* to be concave down. This, in turn, requires the first derivative of *g* to be decreasing.

$$\begin{array}{l} g'(x)=f(x). g'(x)>0 \Rightarrow x\in(-5,-3)\cup(-3,2) \\ g''(x)=f'(x). g''(x)<0 \Rightarrow x\in(-5,-3)\cup(0,4) \end{array}$$. So the answer is $(-5,-3)\cup(0,2)$

3c-AB-BC) $h'(x)=\frac{g'(x)\cdot 5x-g(x)\cdot 5}{(5x)^2}$. So $h'(3)=\frac{g'(3)\cdot(15)-g(3)\cdot 5}{15^2}=\frac{-30-45}{15^2}=-\frac{1}{3}$.

3d-AB-BC) $p'(x)=f'(x^2-x)\cdot(2x-1)$. $p'(-1)=f'(2)\cdot(-3)=-2\cdot(-3)=6$.

So the slope of the tangent line at (-1,0) to the graph of *p* is 6.

One possible scoring:
- 3a) +1 answer
- 3b) +2 interval; +1 explanation
- 3c) +1 h'(x); +1 answer
- 3d) +2 p'(x); +1 answer

AB4+BC4) Train A velocity given by $v_A(t)$, measured in meters per minute, *t* is measured in minutes. Selected values in table.

4a-AB-BC) Average acceleration = $\frac{v_A(8)-v_A(2)}{8-2}=\frac{-120-100}{6}=\frac{-220}{6}=-\frac{110}{3}$ meters per minute per minute.

4b-AB-BC) Train A's velocity will be -100 meters per minute at some point on the interval $(5,8)$. The velocity is given by a differentiable function, and so the velocity is continuous. Since the velocity is continuous, the Intermediate Value Theorem (IVT) guarantees that there is a c in $(5,8)$ with the velocity equal to any intermediate value. Since the velocity decreases from 40 to -120 during the interval $(5,8)$, and -100 is between -120 and 40, then there is a c in $(5,8)$ with a velocity of -100.

4c-AB-BC) position at t=2 minutes is +300 meters (300 meters east of Origin Station).

Position at t=12 minutes = position at t=2 minutes + total change in position over the interval (2,12). This is:

$$300+\int_{2}^{12} v_A(t)dt \approx 300+\left[\frac{1}{2}(5-2)(100+40)+\frac{1}{2}(8-5)(40+(-120))+\frac{1}{2}(12-8)(-120+(-150))\right]=-150$$

So the position of the train will be approximately 180 meters west of Origin Station.

4d-AB-BC) We know that the square of the distance between train A and train B is the sum of the squares of the distances of train A and train B from Origin Station. So –

$d(t)^2 = p_A(t)^2 + p_B(t)^2 \Rightarrow 2d(t)d'(t) = 2p_A(t)p_A'(t) + 2p_B(t)p_B'(t)$. So

$$d(t)d'(t) = p_A(t)p_A'(t) + p_B(t)p_B'(t) \Rightarrow 500d'(t) = 300(100)+400\left(\frac{d}{dt}\left(p_B'(t)\right)\Big|_{t=2}\right)=$$

$$300(100)+400\left(\left(-5t^2+60t+25\right)\Big|_{t=2}\right)=30000+400(125)=80000$$

So $d'(t)=\frac{80000}{500}=160$ meters per minute.

One possible scoring:
- 4a) +1 setup and answer
- 4b) +2 answer with explanation
- 4c) +1 integral expression; +1 trapezoidal sum; +1 answer
- 4d) +1 distance; +1 derivative w.r.t. t; +1 answer

AB5) *f* and *g* are twice-differentiable (and therefore continuous). Values and signs for various values are in a table.

5a-AB) Relative minima of *f* can be identified when the first derivative of *f* changes from negative to positive. This happens (in [-2,3]) when *x = 1*.

5b-AB) $f'(-1)=f'(1)=0$. So the average rate of change of the velocity from t=-1 to t=1 is $\frac{0-0}{1-(-1)}=0$. Since *f′* is continuous (it is differentiable and therefore continuous), the Mean Value Theorem (MVT) guarantees a *c* in (-1,1) with the derivative of *f′* = 0. This means at that *c*, *f′′(c)=0.*

5c-AB) $h'(x)=\frac{f'(x)}{f(x)} \Rightarrow h'(3)=\frac{f'(3)}{f(3)}=\frac{0.5}{7}=\frac{1}{14}$.

5d-AB) by the FTC, $\int_{-2}^{3} f'(g(x))\cdot g'(x)dx = f(g(3))-f(g(-2))=f(1)-f(-1)=2-8=-6$

One possible scoring: 5a-AB) +1 x value; +1 explanation
5b-AB) +1 continuous; +1 MVT
5c-AB) +1 derivative; +1 answer
5d-AB) +1 FTC; +1 intermediate values of g; answer

BC5)

5a-BC) Area= $\int_0^1 xe^{x^2} - (-2x)dx = \left(\frac{1}{2}e^{x^2} + x^2\right)\Big|_{x=0}^{x=1} = \frac{e}{2} + 1 - \frac{1}{2} = \frac{e+1}{2}$

Note: for the first term, use the substitution $u = x^2$

5b-BC) $Volume = \pi\int_0^1 \left(xe^{x^2} + 2\right)^2 - (-2x+2)^2 dx$

5c-BC) $Perimeter = e + 2 + \int_0^1 \sqrt{1+\left(e^{x^2} + 2x^2e^{x^2}\right)^2} dx + \int_0^1 \sqrt{1+(-2)^2} dx$

Note: 5a-BC and 5b-BC will be used for the AB subscore.

One possible scoring: 5a-BC) +1 integrand and limits; +1 antiderivative; +1 answer
5b-BC) +2 integrand; +1 constant and limits
5c-BC) +1 length of vertical segment; +1 length of upper boundary; +1 length of lower boundary

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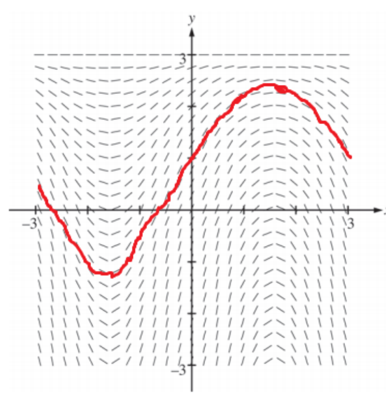
AB6) $\frac{dy}{dx} = (3-y)\cos x$. Let $y = f(x)$ be the particular solution to this differential equation with initial condition $f(0) = 1$

6a-AB) Bad graph of resulting slope field:



6b-AB) At (0,1) the slope is: $\frac{dy}{dx} = (3-y)\cos x = (3-1)\cos(0) = 2$. So the tangent line is: $y - 1 = 2(x-0)$. Then we have: $y \approx 1 + 2(0.2) = 1.4$.

$$\frac{dy}{dx} = (3-y)\cos(x) \Rightarrow (+1) \Rightarrow \frac{dy}{3-y} = \cos(x)dx \Rightarrow (+3) \Rightarrow -\ln\left|(3-y)\right| = \sin(x) + C$$

$$f(0) = 1 \Rightarrow (+1) \Rightarrow -\ln(2) = C = \ln\left(\frac{1}{2}\right)$$

6c-AB)

$$\ln\left|3-y\right| = -\sin(x) - \ln\left(\frac{1}{2}\right)$$

$$(+1)\ \left|3-y\right| = \frac{1}{\frac{1}{2}e^{\sin(x)}} = \frac{2}{e^{\sin x}}.$$

We want the curve that contains (0,1); so we have $y = 3 - \frac{2}{e^{\sin x}}$

One possible scoring: 6a-AB) +1
6b-AB) +1 tangent line; +1 approximation
6c-AB) +6 as indicated

BC6) Taylor Series for $f(x)$ about *x = 1* is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n}(x-1)^n$. Converges to $f(x)$ for $\left|x-1\right| < R$.

6a-BC) $\lim_{n\to\infty}\left|\frac{2^{n+1}}{n+1}(x-1)^{n+1}\cdot\frac{n}{2^n(x-1)^n}\right| = \lim_{n\to\infty}\frac{2n}{n+1}\left|x-1\right| = \left|x-1\right|\lim_{n\to\infty}\frac{2n}{n+1} = 2\left|x-1\right|$

The ratio test says that $2\left|x-1\right|$ must be less than 1; so we have $\left|x-1\right| < \frac{1}{2}$ and the value of R is $\frac{1}{2}$.

6b-BC) $f'(x) = \sum_{n=1}^{\infty}(-1)^{n+1}2^n(x-1)^{n-1} = 2 - 4(x-1) + 8(x-1)^2 - .... + 2(2-2x)^{n-1} + ...$

6c-BC) $f'(x) = 2\sum_{n=0}^{\infty}(2-2x)^n$. We know that $\sum_{n=0}^{\infty}(x)^n = 1 + x + x^2 + ... = \frac{1}{1-x}$ so, replacing 'x' with '2-2x' and multiplying by 2 yields $f'(x) = \frac{2}{1-(2-2x)} = \frac{2}{2x-1}$. So $f(x) = \int\frac{2dx}{2x-1} = \ln\left|2x-1\right| + c$.

We know that $f(1) = 0$ so *c = 0* and $f(x) = \ln(2x-1)$. We can dispense with the absolute values because *2x-1* will always be positive within the radius of convergence.

One possible scoring: 6a-BC) +1 ratio test; +1 answer
6b-BC) +1 derivative; +1 first 3 terms; +1 general term
6c-BC) +2 identifying f'(x); +1 antiderivative +c; +1 solve for c

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Comments and questions to Bob Wilder, bob.wilder@appo.k12.de.us